Numerical simulation of HTS dc-SQUID by FEM coupling with circuit equation taking into account phase difference of Josephson junction

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An HTS SQUID is a high sensitive magnetic sensor. The SQUID has been widely studied for various applications. In those applications, magnetic shielding environment is required for the stable operation of SQUID. The reason why the magnetic shielding is employed is that the SQUID is adversely affected by magnetic noise. Because of the background, the high robustness with respect to magnetic noise is needed for the SQUID. To attain the SQUID operation without magnetic shielding environment, it is necessary to clarify the phenomenon inside of the SQUID and improve the performance. However, the phenomenon such as quantum and electromagnetic behaviors has not been comprehended yet. In our study, the FEM is employed to simulate the electromagnetic behavior of the SQUID and coupled with the equivalent circuit of the Josephson junction to consider the quantum behavior. By using the proposed simulation method, the numerical simulation of SQUID is performed taking into account both of the electromagnetic and quantum behavior. In the paper, the characteristics of SQUID that is affected by the amount of the external magnetic flux are shown.

***Index Terms*— Electromagnetic field simulation, Josephson effect, Josephson junction, quantum magnetic simulation, SQUID.**

I. Introduction

HIGH TEMPERATURE SUPERCONDUCTING (HTS) quantum interference device (SQUID) is a superconducting device and utilizes signature phenomenon such as quantization of magnetic flux and Josephson effect. The SQUID has been developed as an extremely sensitive superconducting magnetic sensor. In recent, it has been widely studied for various applications, e.g., a biomedical device [1], a low field magnetic resonance imaging (MRI) [2], and so on. For these applications, the SQUID is used in the magnetic shielding environment. It is because that the SQUID is easily influenced by small magnetic noise and becomes unstable. Therefore, magnetic shielding environment is required for the SQUID to perform the stable operation, and it interrupts the broad use of the SQUID.

In our study, the final goal is to realize a stable operation of a SQUID magnetometer, which is a kind of SQUID, even without magnetic shielding environment. To achieve the goal, it is essential to improve the sensitivity to magnetic noise. The operation mechanism of the SQUID is microscopic phenomenon, whereas the SQUID is a macro-scale device. Therefore, we have to consider both the micro and macro phenomenon in order to accurately simulate the SQUID operation. At first, we have previously performed the electromagnetic field simulation of the SQUID magnetometer in the full model [3], and the finite element method (FEM) was employed. In the simulation, only the macroscopic phenomenon is expressed well. However, the characteristics of the Josephson junction were not considered yet. Subsequently, the characteristics of Josephson junction were microscopically considered by using equivalent circuit [4]-[5]. However, the characteristics of the SQUID, such as voltage-magnetic field characteristic, were not completely obtained.

In this paper, the simulations of SQUID are performed in the various external magnetic fields. From the simulation results the temporal average voltage is obtained, and it is a major characteristic of SQUID.

II. DC-SQUID

A simple dc-SQUID is considered as a simulation model. Fig. 1 shows the shape of simple dc-SQUID.

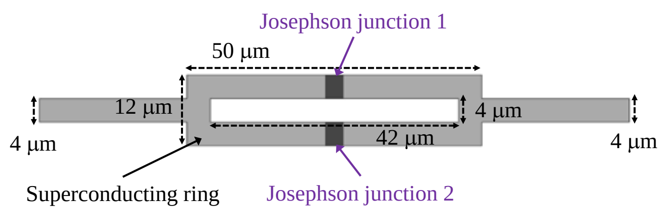

Fig. 1. Geometric sketch of simple dc-SQUID model.

As shown in Fig. 1, the simple dc-SQUID model consists of a superconductor ring and two Josephson junctions. The Josephson junction shows the nonlinear current-voltage characteristic due to microscopic phenomenon. The Josephson junction can be expressed by using an equivalent circuit called the RSJ model [6]. Fig. 2 shows the equivalent circuit of the dc-SQUID with two Josephson junctions, and the circuit equations of the dc-SQUID are given by (1) – (4). Furthermore, the quantization of magnetic flux at the dc-SQUID is given by (5).

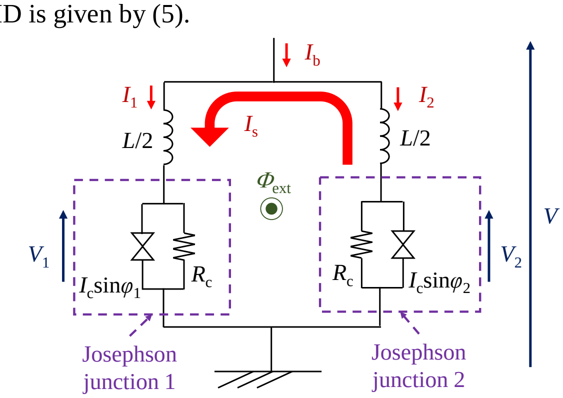

Fig. 2. Equivalent circuit of dc-SQUID.

$$I_1 = \frac{I_b}{2} - I_s = I_c \sin\varphi_1 + \frac{V_1}{R_c} \quad (1)$$

$$I_2 = \frac{I_b}{2} + I_s = I_c \sin\varphi_2 + \frac{V_2}{R_c} \quad (2)$$

$$V_1 = \frac{\Phi_0}{2\pi}\frac{d\varphi_1}{dt} \quad (3)$$

$$V_2 = \frac{\Phi_0}{2\pi}\frac{d\varphi_2}{dt} \quad (4)$$

$$\mathit{\Phi} + \frac{\mathit{\Phi}_0}{2\pi}(\varphi_1 - \varphi_2) = n\Phi_0 \quad (5)$$

where I_b, I_1, I_2, I_c, I_s, V, V_1, V_2, φ_1, φ_2, L, R_c, $\mathit{\Phi}$, $\mathit{\Phi}_{ext}$, Φ_0, and t are the bias current, the junction currents 1 and 2, the critical current, the screening current, the SQUID voltage, the junction voltages 1 and 2, the gauge-invariant phase difference of the wave function at junctions 1 and 2, the inductance of SQUID ring, the characteristic resistance (3.75 mΩ constant in this paper), the magnetic flux crossing the dc-SQUID, the external magnetic flux, the flux quantum, and the time, respectively. Here, $\mathit{\Phi}$ and V are given by

$$\mathit{\Phi} = \mathit{\Phi}_{ext} + LI_s \quad (6)$$

$$V = \frac{V_1 + V_2}{2} = \frac{\Phi_0}{4\pi}\frac{d(\varphi_1 + \varphi_2)}{dt}. \quad (7)$$

In the SQUID, the temporal average of the voltage $\tilde{V}$ is experimentally observable, and it is related to an external magnetic flux measured.

$$\tilde{V} = \frac{1}{T}\int_0^T V dt \quad (8)$$

where T is the time period. In this paper, the relation of temporal average voltage and external magnetic field is simulated.

III. Simulation Method and Results

In the developed method, the FEM is a fundamental simulation method. Additionally, the circuit equations (1) – (4) and the equation of the flux quantization (5) are embedded to consider the characteristics of the Josephson junctions and the flux quantization. These equations can express the changing of the Josephson junction voltage with the time even though the external magnetic field is constant. The detail of the simulation method will be explained in the extended paper due to lack of space. In this paper, the simple SQUID is simulated under a few difference external magnetic field conditions.

As a simulation results, Figs. 3 and 4 show the time transitions of the voltage and the gauge-invariant phase difference. In Figs. 3 and 4, three cases of the external magnetic flux, 0.1 Φ_0, 0.2 Φ_0, and 0.3 Φ_0, are shown. In Fig. 3, the shorter the time period of the SQUID voltage is, the higher magnetic flux is applied. From Fig. 4, it is obvious that the increment ratio of the phase difference increases with the increase of the external magnetic flux. Table I shows the relation of the time period and the temporal average of the voltage values, that is the principal characteristic of dc-SQUID. From the results, it is clear that the temporal average of the voltage is linearly large with the increase of the external magnetic flux. Through the developed simulation, the SQUID characteristics affected by the amount of the external magnetic flux are confirmed. In the extended paper, more various conditions will be simulated and the $\mathit{\Phi}$-V characteristics of the SQUID will be also shown.

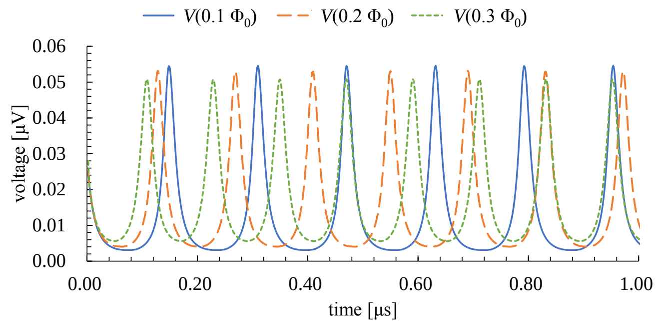

Fig. 3. Time transition of SQUID voltage

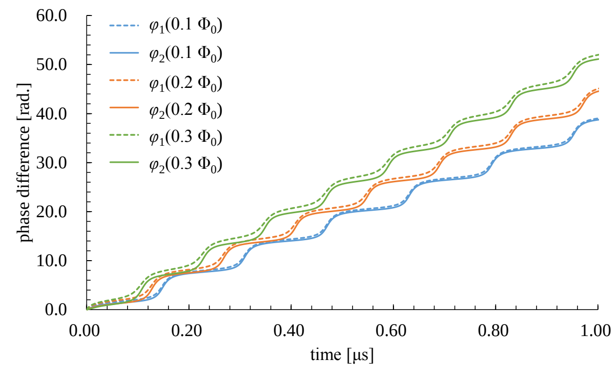

Fig. 4. Time transition of gauge-invariant phase difference

TABLE I
Time Period and Temporal Average of Voltage

External magnetic flux [Wb]	Time period [μs]	Temporal average of the voltage [μV]
0.1 Φ_0	0.160	0.0130
0.2 Φ_0	0.140	0.0149
0.3 Φ_0	0.122	0.0178

IV. References

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